Richard Streit Hamilton (1943–2024)

Simon Brendle, Bennett Chow, and Panagiota Daskalopoulos

Richard Streit Hamilton was born in Cincinnati, Ohio, on January 10, 1943, the younger of two sons of William Selden Hamilton and Hester Hamilton (née Streit). His father was a surgeon, who served in the United States Navy during the first two years of Richard's life. His mother was a homemaker. Richard attended Lotspeich Elementary School and Walnut Hills High School in Cincinnati. In 1959, at the age of 16, Richard enrolled at Yale University, graduating summa cum laude in 1963. Richard then moved to Princeton, where he received his PhD under the direction of Robert Gunning in 1966.

Richard's first academic position was at Cornell University, where he worked with James Eells, Jr. In 1982, during his years at Cornell, Richard published his landmark paper on the Ricci flow. In this work, Richard introduced a novel tool to differential geometry—a nonlinear heat equation for Riemannian metrics. Even more remarkably, he proved several ingenious estimates for this new flow, which allowed him to answer a deep question in differential geometry—the classification of compact three-manfolds admitting metrics with positive Ricci curvature. Following his initial breakthrough on the Ricci flow, Richard moved to San Diego, where he achieved a number of major advances over the years. Some of the milestones include Richard's 1986 paper, which developed a general framework for curvature pinching in arbitrary dimension; his 1988 work on the Ricci flow in dimension 2, giving a new proof of the classical uniformization theorem; and his striking breakthrough in 1993 on the

Figure 1. A childhood portrait of Richard and his older brother Billy, painted by their mother Hester.

Simon Brendle is a professor of mathematics at Columbia University. His email address is simon.brendle@columbia.edu.

Bennett Chow is a professor of mathematics at the University of California at San Diego. His email address is bechow@ucsd.edu.

Panagiota Daskalopoulos is a professor of mathematics at Columbia University. Her email address is pdaskalo@math.columbia.edu.

Communicated by Notices *Associate Editor Daniela De Silva.*

DOI: https://doi.org/10.1090/noti3212

matrix Harnack inequality for the Ricci flow, which later came to play a central role in Grigori Perelman's work.

Prior to Richard's work on the matrix Harnack inequality, Li and Yau had succeeded in proving a differential Harnack inequality for positive solutions of the linear heat equation on manifolds with positive Ricci curvature. One of the difficulties Richard faced was that the Ricci flow is not a scalar equation, but rather a system of nonlinear heat equations. Richard showed that every solution to the Ricci flow with positive curvature satisfies a tensorial Harnack inequality, which makes it possible to compare the curvature across different points in spacetime. To that end, Richard introduced a carefully chosen tensorial quantity involving the curvature tensor and its first and second covariant derivatives. This was a stunning leap.

In the late 1990s, Richard left California for Columbia University in New York City. By that time, Richard had developed a program aimed at proving the Poincaré conjecture by means of Ricci flow with surgery. A major difficulty at the time was to rule out collapsing. In a stunning series of breakthroughs, Perelman in 2002 succeeded in proving a noncollapsing estimate for the Ricci flow, and in 2003 gave a proof of the Poincaré conjecture. Perelman's monumental proof built on many deep ideas Richard had developed over the years, combined with many brilliant new ideas introduced by Perelman.

Richard's pioneering work has opened up a new field of geometric flows. He has equipped mathematicians with new tools to take on fundamental questions in geometry and topology that had been intractable before. His groundbreaking contributions have been recognized with numerous awards including the Oswald Veblen Prize (1996); the Clay Research Award (2003); the Steele Prize for Seminal Contribution to Research (2009); the Shaw Prize (2011); and the Basic Science Lifetime Award in Mathematics (2024). Richard was elected a member of the National Academy of Sciences (1999) and the American Academy of Arts and Sciences (2003).

Richard pursued his passion for mathematics throughout his life, inspiring generations of geometers. He passed away on September 29, 2024, in New York City.

Steven Altschuler and Lani Wu

We had the privilege of being Richard's PhD students while he was at the University of California at San Diego (UCSD). We remained friends long after graduating. In recent years, we often met around Thanksgiving in New York City at his favorite restaurant, where the staff knew him by name, and then strolled through Central Park talking about, well, everything. This year, as we walked through the park, looking at the beautiful fall trees that Richard loved so much, we thought about the profound impact he had on our own journey through mathematics, engineering, and life sciences.

Richard taught us how to approach the life cycle of research. Richard chose problems that were beautiful and important. He sketched plans for why a result should be true, then "computed first and asked questions later." If his plans failed, he derived inspiration from adjacent problems—Ricci flow, mean curvature flow, and Yamabe flow were in constant rotation. And, when he solved a problem, his papers and lectures were meticulously crafted to highlight the beauty of the problem, his main insights, and his key calculations. Richard's approach to research has guided us throughout our careers, as useful for attacking mathematics as uncovering mechanisms of human diseases.

Perhaps the most important lessons we learned from Richard were to find joy in everything we do and to do the things that bring us joy. Richard was one of the hardest working people we ever met, though he claimed he never worked a day in his life. Beyond research, Richard showed us the importance of taking time to enjoy the wonderful world around us. When we think back on all that we learned, there were solitons, Harnack estimates, and maximum principles, but there was also windsurfing, snorkeling, horseback riding, and barrel rolls in gliders. One day, visiting Richard in Hawaii, he told us to always acknowledge when we had a perfect day. We are blessed to have been Richard's students and friends, and to have shared many perfect days with him.

Simon Brendle

Richard's work has been a great source of inspiration to me. Indeed, Richard's groundbreaking paper on three-manifolds with positive Ricci curvature was the first research paper in differential geometry I read. I first met Richard in February 2002 at Columbia. During my years at Princeton and Stanford, I returned to Columbia frequently to visit Richard. Each time I came to visit him, Richard was cordial and generous, freely sharing his insights on mathematics and on life. The best time to find him was in the afternoon, after he had finished teaching his class. Sometimes we would discuss in his office all afternoon. On occasion, Richard would show me a calculation he had done with the help of "Muffin," his computer. When it was sunny outside, Richard would invite me on a walk through Riverside Park and we would continue to talk in his favorite cafe on 83rd Street or enjoy a picnic in the park. When Richard was doing mathematics, he was full of energy and enthusiasm. Richard had his own way of thinking, making deep ideas appear natural and

elegant. Richard set high standards for himself, and he was not easily discouraged. When a problem seemed out of reach, Richard kept pushing forward anyway. Over dinner, Richard liked to share stories about whale watching off the shore of California, about sea turtles and dolphins near his beach house in Haleiwa, Hawaii, or about riding his horse in the mountains near San Diego. Richard had a profound knowledge of ancient Greek and Roman history. One of his favorite stories concerned Queen Artemisia and the battle of Salamis; another favorite story of his concerned the life of Cincinnatus, the namesake of Richard's hometown. After I moved to Columbia permanently, I had the great privilege of sitting in Richard's topics course on Ricci flow for one semester, where Richard lectured on the universal pinching estimate in dimension 3, his famous matrix Harnack inequality, and the nonconic estimate, among other topics.

Richard forged paths where nobody had gone before and made discoveries that nobody thought possible before. I will never forget Richard's generosity, his sense of humor, and his lifelong passion for mathematics.

Figure 2. Richard riding his horse "Mystic Mercedes" near San Diego.

Robert Bryant

My first experience of Richard Hamilton's mathematics was reading his two beautiful 1982 articles, "The inverse function theorem of Nash and Moser" (*Bulletin of the AMS*) and "Three-manifolds with positive Ricci curvature" (*Journal of Differential Geometry*). I was an assistant professor at Rice when they appeared, and his masterful exposition of the Nash-Moser theory was a revelation. I had been thinking about isometric embedding problems and Cartan-Kähler theory and, in the back of my mind, hoping that understanding Nash-Moser better would help in other geometric PDE problems. His brilliant use of the Ricci flow and his celebrated theorem that it evolves compact three-manifolds of positive Ricci curvature to ones of constant sectional curvature were awe-inspiring.

I first got to spend time with Richard in 1985, when Shiing-Shen Chern invited us both to a two-week summer conference at Fudan University in Shanghai. It was the first time that either of us had been to China, and our very kind hosts saw to it that we foreign guests had plenty of opportunities to explore cultural sites, along with social events and banquets, in which we got to mingle and interact. Richard was the perfect traveling companion, always in good humor and full of great stories. I was delighted to learn that, in addition to mathematics, we both shared a love of ancient Greek literature. Unlike me, though, Richard had read them in the original Greek, and he had an astonishing recall of details about the translations that would make for many amusing and enlightening conversations over the course of our friendship. During that time, Richard suggested some interesting problems involving invariants of coframings of three-manifolds to work on, and that had a profound influence on my early work on geometric PDE in dimension 3, such as the prescribed Riemann curvature tensor problem, among others.

After I moved to Duke in 1987, Richard's brother William ("Billy") was on the faculty at Wake Forest University, and Richard always made it a point to let me know when he was coming to North Carolina to visit family. He would come by to visit me at Duke, and we would talk about problems we were working on or thinking about. I remember especially one visit when, after a morning of talking about a geometry problem, Richard wanted to do something active, so he asked where one might go water skiing. I didn't know (having never been on water skis), but I thought there might be something at nearby Jordan Lake. So off we went, found a boat and skis to rent at the Jordan Lake marina, and Richard spent the afternoon teaching me water skiing. It was the kind of spontaneity that Richard was famous for, and why I always remember Richard's visits with special fondness.

It was during one of these visits that Richard talked about the problem of understanding solitons for the Ricci flow in dimension 3. Of course, he had already found his famous "cigar soliton" in dimension 2, but the question of classifying the solitons in dimension 3 was still open. After the visit, I figured out that there was an essentially unique complete, rotationally invariant steady Ricci soliton in dimension 3 and wrote a letter to Richard describing the proof. (This was before we all started using TeX, so it was a handwritten letter.) Richard started calling it the "Bryant soliton" after that. I never did get around to writing up my existence argument until, several years later,

Perelman's work came out and people started asking where they could see the details. At that point, Richard couldn't find my original letter, so I had to reconstruct the argument from memory.

Richard was always passionate about what he did, whether it was mathematics, surfing, horseback riding, or opera, and he generously shared his excitement with those around him. He was not just a great and inspiring mathematician, but a friend who enriched the lives of all around him. I feel especially lucky to have had the chance to know and work with him.

Huai-Dong Cao

On November 5, 2024, I flew to Cincinnati, a city full of Richard's fond memories of his school years, his parents, and his son Andrew. Over the years, Richard and I had traveled to many countries together, often joined by my wife, Belinda. Sadly, this time I was traveling alone for Richard's burial service, which had been planned for the next day by Andrew and Susan Harris, Richard's longtime partner. Although over a month had passed since I first learned from our friend Toti Daskalopoulos about Richard's fall, and then his passing a week later, I still had a hard time accepting that he had left us. Just months earlier, Richard and I had spent over a week together in Beijing at the 2024 International Congress of Basic Science, where he received the Basic Science Lifetime Award in Mathematics from Yau. Needless to say, the flight to his hometown for his burial brought back a flood of memories.

I first met Richard in 1982 as a first-year graduate student at Princeton. It was at a lecture organized by Yau, my PhD adviser, at the Institute for Advanced Study. Richard presented his seminal work on the Ricci flow which attracted a large audience. Knowing almost nothing about basic differential geometry, I hardly understood anything from the talk. To my surprise, shortly after Richard's visit, Yau asked Ben Chow, a fellow first-year student, and me to study Richard's paper carefully and present it at the weekly learning seminar. That very seed planted by Yau would eventually blossom into a profound lifetime friendship, more than just a mathematical kinship, between Richard and me.

My memory also took me back to UC San Diego. Yau moved there in Fall 1984 and soon brought Rick Schoen, Richard, and Gerhard Huisken on board. It was a paradise for us to learn geometric analysis. By then, I had grasped Richard's 1982 *Journal of Differential Geometry* (JDG) paper of the Ricci flow on three-manifolds and completed my thesis on the Kähler-Ricci flow under Yau's guidance. I saw Richard regularly and also met Susan for the first time on UCSD's campus. One day, Richard asked me to explain a recent result in the field. He listened quietly, seemingly absent-minded. Then, after a long silence, he walked to the blackboard and sketched an entirely different proof of a key lemma. Later, he shared his method: "When I hear a new result that interests me, first I ask myself if I believe it and then start to think if there is a possible counterexample. If I can't think of one, then I ask myself how I would prove it." I was amazed, never imagining before that one can learn other people's work by intentionally not going through their proofs! Better yet, Richard liked to share his brilliant mathematics with anyone interested in learning his work. He loved to do that, explaining in detail his discoveries or new ideas that had recently captured his interest. One day, I asked how he would judge good or bad math. His answer: "I don't judge it as good or bad. What matters to me is if it is interesting or not." That simple line seems to me like Richard's manifesto. As a great mathematician of our time, he was motivated by such a pure, clean, and lasting drive—a genuine love and interest in the mathematics topic of his choice.

Richard gave credit to his luck including running into Yau, who saw the vast potential of the Ricci flow from the very beginning and supported him strongly all along. If luck and interest let Richard travel great miles in his pursuit of the Ricci flow, then his kindness and generosity touched the deepest chord of people around him, both in mathematics and in life. From the mid-1980s to 1990s he attracted a number of brilliant minds, like Gerhard, Tom Ilmanen, Ben Andrews, and others, and formed a "principality" of geometric flows within the field of geometric analysis founded largely by Yau. It was a pure and harmonious "principality," where new ideas were exchanged freely. Richard very much cherished that period even decades later.

Before the Covid-19 pandemic, Richard liked to visit us in the Lehigh Valley from New York City, especially during winter breaks. After dinner, Richard and I often lit the fireplace. Then Richard would lie on the carpet in front of the flames and ask Belinda to play Tchaikovsky, Piano Concerto No.1, and start telling us interesting stories. Among all the stories he told us, one captured a facet of his character. During the long and harsh winters at Cornell, he liked to keep the fire roaring in the fireplace. One day Richard was home alone when somehow the flames suddenly spread outside the fireplace. Panicked, Richard called his landlady to borrow a fire extinguisher. To his surprise, her young niece answered. "She was a true beauty!" And despite smoke filling the room, Richard managed to chat with the young woman for several minutes before finally asking, "Can I borrow your fire extinguisher?" "What for?" she asked. "There is a fire in my room!" "Ah!" Richard's impersonation of her shocked gasp had us all bursting into laughter.

That is the Richard I remember, who brought us so much joy, wisdom, and inspiration.

"Death, be not proud"—Tom sent me a poem by John Donne from Zürich after I shared with him the news of Richard's passing, an unbearable loss we must endure in years to come. Yet, our fond memories of him will always bring him back, at least in spirit.

Figure 3. Richard receiving the Basic Science Lifetime Award in Mathematics in Beijing, 2024.

Bennett Chow

I have been fortunate to have known Richard for more than forty years. He was an exceptional teacher, mentor, and friend. He was consistently good-spirited, humorous, optimistic, and generous in all our interactions. I would like to share some of the teachings and advice he gave me.

One of the most striking aspects of Richard's approach to mathematics was the joy he conveyed during the process. He was always actively thinking and coming up with new and original ways to look at things. Although Richard loved surfing and horseback riding, mathematics was just as relaxing to him.

Another striking aspect of Richard's approach to mathematics was his fearlessness and willingness to think on a big scale. I recall him once quoting Mike Freedman, who likened doing mathematics to being in a dense forest with only a machete, pursuing a rumor of hidden gold. This quote epitomizes Richard's approach to mathematics. Richard's advice was to work on the hardest part of a problem first. He also said it was just as easy to fail at a minor problem as at an important one, so one might as well work on the important ones.

A few years ago, when I asked Richard for research advice, he told me to approach it with energy and enthusiasm. This was one of the secrets to his success. Richard viewed research as a joyful opportunity and never hesitated to forge ahead.

Richard was extraordinarily generous with his mathematical ideas. I often had the privilege of learning about his ideas long before they were published. The mathematical community has also benefited immensely from his generosity. He did mathematics not to seek credit, but rather, as Richard Feynman would say, "for the pleasure of finding things out."

Richard was a remarkably humble person. He would often credit his success to luck, while it was clear to the rest of us that it stemmed from his brilliance and deep thinking.

Richard is deeply missed by all who had the privilege of knowing him. His spirit lives on through his contributions and the countless lives he influenced.

Demetrios Christodoulou

I first met Richard in 1986, when I was on leave from Syracuse University to UCSD. At that time S.T. Yau had turned the UCSD Mathematics Department into the leading center for geometric analysis in the world, with many luminaries around. It was a wonderful time. This was the time when the full outlines of Richard's approach to the Poincaré conjecture became clear in his mind. From the beginning I was attracted by Richard's directness of character as well as by the freshness and boldness of his approach to mathematical problems, and we became friends. Later on I moved to Courant and then, in 1992, to Princeton. That year there was a special program at the Institute for Advanced Study on geometric flows, focused on Hamilton's Ricci flow, and Richard was naturally the main lecturer. This gave me another long-term opportunity to interact with him. I discovered then, to my amazement, that he had a profound knowledge of ancient Greek literature. As he explained, he had first studied ancient Greek literature in the original at Yale at age 16, before focusing on mathematics. Some years later, the possibility arose of a professorship offer to Richard from Princeton. I was asked to present his work to the department. The topologists were skeptical and warned me not to mention anything about a possible connection to the Poincaré conjecture. I replied that I do not talk about topics of which I have no expertise and that his work on geometric analysis more than suffices. The offer to Richard did materialize, but, in 1998, he went to Columbia instead. In the period when he was at Columbia and I was at Princeton we got together many times, in New York as well as at Princeton. Most memorable was his last visit to Princeton while I was

still professor there, in May 2001. We stayed at Alchemist & Barrister until the wee hours of the morning discussing mathematics and life. Later that day I took the flight of my return to Europe. In the spring of 2011 Richard and I had a wonderful time together in China, visiting Tsinghua University in Beijing. One morning in June of that year, as I came to my office at ETH in Zürich, I saw the message from Hong Kong that I was to share with Richard the 2011 Shaw Prize in Mathematical Science, which naturally made me very happy. The next day though I received another message from Hong Kong saying that they had been trying to contact Richard to tell him the good news, but had been unable to reach him. I knew that at that time of year he would be in Hawaii, probably out on a boat trip. He had given me his personal email there, so I first got in touch with him and passed the message from Hong Kong. He replied to me with the ancient proverb: "Fortune is a goddess who favors the brave."

Figure 4. Richard encountering a shark while swimming in the ocean near his house in Hawaii.

Tobias Holck Colding

The first time I met Richard, I was an assistant professor at the Courant Institute in New York City. Richard had come to the Courant Institute for the day to learn as much as he could from Louis Nirenberg and Luis Caffarelli about Harnack inequalities. It was the fall of 1995 and Caffarelli had joined the faculty at NYU the year before. On that day, Sylvain Cappell asked me to join him and the other three for lunch at the Violet. The Violet was then a popular cafe on the corner of Washington Square just across the street from the Courant Institute. I watched and listened with fascination as Richard scribbled down possible inequalities on the white paper tableclothes in the restaurant. Richard was hoping that Luis or Louis would point him in the direction of something useful. He felt certain that Harnack inequalities should play an essential role in his program for the Ricci flow. Richard had then already published his matrix Harnack inequality for the heat equation, as well as a couple of articles about Harnack inequalities for Ricci flow and he felt that more should be true. Time proved him right about the importance of the Harnack inequality in his program.

Through the years, I got to know Richard. It was always a treat to see him and something that I was always really looking forward to. To hear his thoughts on problems and questions was always very enlightening and he always had something interesting to say. It was clear that he had already thought deeply about many of the questions that came up. Richard had broad interests outside of mathematics. For a time, he had season tickets to the Metropolitan Opera in New York City and invited me to join him in his box for an opera. We had a very memorable evening. Even though the performance was great, the most memorable part of the evening was the time we were talking during the intermission. Later I spent a semester at Columbia University and he graciously offered that we could share his office. Richard was an analyst at heart, but he had fantastic geometric ideas. He gave spectacular lectures that were extremely inspiring. He was also a very gifted expository writer. His extraordinary vision and insight for mathematics, as well as his generosity and kindness as a human being, will be sorely missed.

Panagiota Daskalopoulos

It is with deep sorrow that I sit down to write my recollection of the life of our colleague at Columbia University and my dear friend for many years, Richard Hamilton. My mind travels back to about 30 years ago; it was the Fall of 1992 at the Institute for Advanced Study (IAS) in Princeton, when I met Richard for the first time. I was giving a seminar on my thesis work on Harnack inequalities and Richard came to my lecture. I was a young PhD student and a member at IAS working with Luis Caffarelli. Richard was also a member as part of a special program in geometric analysis. At the tea that followed the lecture, Richard greeted me speaking in ancient Greek. He spoke about his admiration for ancient Greek and Roman philosophy, a passion that he perceived throughout his life. He then briefly spoke about the Ricci flow, concentrating mainly on the analytical challenges of his program. I was immediately impressed by the clarity with which he explained complex mathematical ideas. In discussions that followed, Richard pointed out that the logarithmic fast diffusion equation, that I was currently studying with Manuel del Pino, was equivalent to the two-dimensional Ricci flow. Back then this had just been observed by Sigurd Angenent and Richard's PhD student Lani Wu. There was obviously so much to discuss! This was the beginning of my

collaboration with Richard that had a profound influence on my development as a mathematician. I saw in Richard an intellectual who always viewed things in his own way—from history, music, politics, and especially mathematics.

I continued collaborating with Richard as a young professor at UC Irvine while Richard was a professor at UCSD. I would often drive down to UCSD for the day, which included lunch at Piatti, Richard's favorite restaurant on the La Jolla shore, a walk by the beach, followed by hours of calculations at Richard's office. Working with Richard was always such a pleasure! He was never intimidating to young mathematicians, neither by his knowledge nor by his style of work.

In the late 1990s Richard started visiting Columbia University, often for a semester every year. Being especially fond of Columbia professors Masatake Kuranishi and Duong Phong, he enjoyed his long visits in New York very much. During this time he also had the opportunity to meet his friend S. T. Yau at Harvard. Over the years, Richard often spoke about those many hours of work with Yau with nostalgia and pride. He greatly valued their friendship, which was based on mutual admiration and support. Richard finally decided to move to Columbia University. The dream of professors Phong and Kuranishi was to build a strong geometric analysis group. I felt very honored when I was invited to join this group shortly afterwards.

In the New York years, my friendship with Richard grew. We both shared a passion for music and often attended concerts at Lincoln Center together. For many years Richard had four box seats at the Metropolitan opera, and it was his pleasure to invite his friends to spend the night at the opera. He later became more fond of the Philharmonic and the New York City Ballet. Not having his horse around, Richard started taking long walks in Central Park that would always end at the zoo. Puffins, the beautiful seabirds, and snow leopards were his favorites. He would often send photos to his friends from the park, usually of flowers and his favorite animals from the zoo.

Richard had a deep appreciation for beauty, especially in nature. He was as fond of beautiful sunsets, flowers, and the deep blue of the ocean, as of Beethoven's and Mahler's symphonies (some of his favorites!) and Balanchine's ballets. But this drive for beauty also had a great impact on his mathematics, which is characterized by elegance and simplicity in its profound depth. This was achieved not only because of his brilliant mind, but also because of his genuine love and lifetime devotion to mathematics. From a young age, Richard suffered from allergies. Surfing in Hawaii or riding horses in California were not just a joy for him, but also a means for keeping his mind clear for his ultimate purpose in life: doing mathematics.

Despite some recent health challenges, Richard visited China in the summer of 2024 to receive the Basic Science Lifetime Award in Mathematics at the International Congress of Basic Science. This was the last time that Richard saw his lifetime friend S. T. Yau. He was proud to receive this prize and he felt very lucky that he went to China to receive this honor.

On Sunday, September 22, 2024, at 6 a.m. in the morning, I received a phone call from Mount Sinai Morningside Hospital. Richard had suffered a severe brain hemorrhage and he was in critical condition. Despite the efforts of his doctors, Richard passed away a week later. A few days before his injury, Richard invited me for a night at the Philharmonic: it was a performance of Mahler's Fifth Symphony, one of his favorites. At the intermission he shared some vivid recollections from his early life: about his son Andrew when he was a young child, and his roommate in college, an accomplished pianist who had a profound influence on his appreciation for music. Before we parted for the evening, he said that his health had improved and that he was excited to share some new mathematical ideas with us in the near future. His eyes suddenly brightened by his enthusiasm and passion for—what else—mathematics!

When I think about Richard now, the following quote attributed to Isaac Newton comes to my mind: "I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me." We deeply miss Richard Hamilton, a giant of mathematics, a critical and original thinker, and above all a dear friend.

Gerhard Huisken

When I first met Richard in 1984, he immediately drew me to the blackboard and began explaining his insights into the special algebraic structure of the Ricci flow, with a particular emphasis on dimension 4. This gave me an early glimpse into his groundbreaking article on metrics with positive curvature operators on four-manifolds, which would later become so influential. That moment was emblematic of Richard's openness in sharing his latest ideas and his remarkable ability to convey the excitement and beauty of the mathematics he had just discovered.

Richard had a strong preference for learning new results directly—whether at the blackboard or during a lecture—rather than through reading papers, whenever possible. This approach often led to fresh perspectives, even for the original authors of the work, as I experienced several times myself. In many cases, he preferred to rediscover results or

find his own proofs rather than searching through the literature.

I fondly recall a wonderfully comical moment during a lecture in Canberra, where Richard explained well-known interior estimates for linear elliptic and parabolic PDEs, all while Neil Trudinger sat in the audience. Through such moments, Richard instilled in his students and junior colleagues a strong sense of confidence in their own ability to "do it yourself."

Richard had a rare gift for combining the beauty of mathematics with the joys of life. Whether it was a walk through nature, a swim in the ocean, a visit to a museum, or a meal at a special restaurant with friends, he celebrated life with the same passion he brought to his mathematics. These encounters, where mathematics was celebrated as a deeply human and enjoyable activity, are what make Richard Hamilton's memory so special to me.

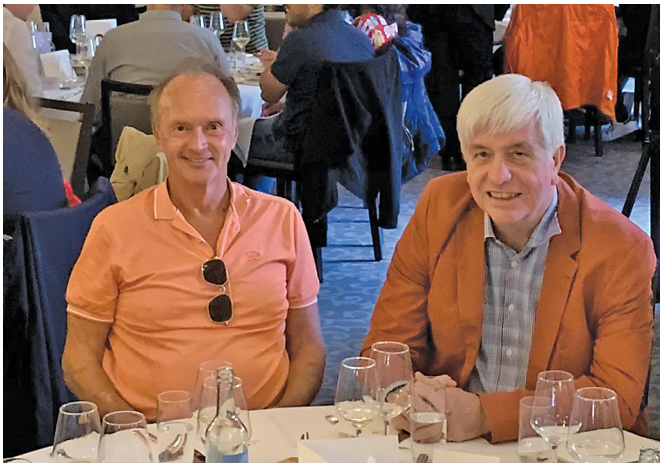

Figure 5. Richard and Gerhard Huisken in Zürich in 2019.

James Isenberg

I first met Richard in 1982. We were both presenting lectures at the Pacific Northwest Geometry Seminar in Vancouver. Richard spoke on his amazing new results which showed that if a compact Riemannian three-manifold has positive Ricci curvature, then the manifold must be the quotient of a three-dimensional sphere. This result was proven using a new technique—the Ricci flow. Although my research specialty was focused on mathematical relativity, I found the mathematics very exciting, largely because of Richard's exciting presentation. A few years later, Mauro Carfora spoke at a gravitational physics conference, and showed how Ricci flow might be very useful in the mathematical study of cosmological solutions of Einstein's equations. Remembering Richard's talk, I began working with Mauro on cosmology and Ricci flow, and this led me to ask Richard if I could come visit him at UCSD for a year in 1989 and learn more about Ricci flow. I was somewhat surprised that he said yes.

That was a wonderful year! Richard and his two graduate students—Steven Altschuler and Lani Wu—and I met for at least an hour every week, talking about all sorts of aspects of Ricci flow, including his recent interest in four-dimensional Ricci flow. Richard told me that the best way to come up with new mathematical ideas was to think about them while you were windsurfing. I was terrible at that, but he told me another way to develop them was to go horseback riding. I was willing to try that out, and so Richard and his girlfriend Susan Harris and my two nieces who lived in La Jolla did that a number of times on the beach right next to the Mexican border. This activity may have contributed to my collaboration with Richard on a paper on "quasi-convergence" of certain Ricci flows.

Richard loved Hawaii. So along with visiting Richard at UCSD, I helped to organize an NSF-CBMS conference in 1989 on "Heat Equations in Geometry" on Oahu, with Richard as the main speaker. There was a lot of beach activity during the day, but most evenings, there were several hours of wonderful discussions of many aspects of Ricci flow. Despite Richard's encouragement, I remained terrible at windsurfing.

Richard continued his spectacular progress in Ricci flow, and since I continued work in that area, we stayed in touch, getting together in San Diego, New York, and Santa Barbara (where we had fun watching polo). Most recently, two years ago I helped organize a conference at the Simons Center for Geometry and Physics at Stony Brook called "40 years of Ricci flow," which marked the work by Richard and many others since 1982 in this very vibrant and mathematically important field. Richard attended the conference, and besides his providing excellent insights into the past, the present, and the future of research on Ricci flow, we had a lot of fun catching up. I miss getting together with Richard very much.

Duong Phong

It is with a profound sense of loss and sadness that I am writing these words in memory of Richard S. Hamilton. I had been a great admirer of his works throughout his career, and it had been my dream to bring him to Columbia University. This dream became a reality in the late 1990s. Ever since, the Columbia mathematics department has had the privilege of basking in his unique vision of geometric analysis, and particularly of the theory of geometric flows that he had created almost single-handedly. Richard and I also became very good friends, beginning with his earliest years at Columbia, where we would go to dinner

together at least once a week at a local Alsatian restaurant. Professor Shing-Tung Yau would join us when he visited Columbia as Eilenberg Chair of Mathematics. It is at these dinners that I learned for the first time about the Li-Yau differential Harnack inequalities and Richard's surgery procedure for the Ricci flow.

I still remember well the first time when Professor Masatake Kuranishi and I brought up the idea of trying to recruit Richard. Because the Columbia Mathematics department is small, there is always a keen competition for positions between different fields, and one never knows how such a suggestion may be received. So we were surprised and delighted when the most senior Columbia faculty member at that time, Professor Lipman Bers, enthusiastically endorsed the idea, and even pointed out the perhaps not so well-known fact that Richard began his career in Professor Bers's field of Riemann surfaces, where he wrote a beautiful thesis. But the main difficulty came from Richard himself, when we approached him; he loved the outdoors, and could not see himself at that time living in a big city. Luckily for us, the situation changed in the late 1990s, and even though Richard had developed at that time a passion for horse riding, he did not see that as incompatible with life in New York City. He found ways of indulging in the region in his earlier activities—Professor Joan S. Birman even tried to help in our recruiting efforts by letting me know that there were horse ranches in the vicinity—and he added new ones: evenings at the opera, enjoying the fall foliage, and the restaurant scene, notably the steak houses, for Richard was unabashedly not a vegetarian. Asked to describe life in New York City, he would give the simple but very perceptive answer: Everyone has his or her own New York City.

Richard's graduate course on geometric partial differential equations became a classic at Columbia. The topics would change from year to year, and cover practically all aspects of the theory of geometric flows. While varied and sometimes involving very different and surprising developments, Richard always strove to present them in his own unique perspective. Thus the participants did not just learn a collection of theorems, as important as they may be, but a whole approach and philosophy of mathematics. Many students and postdoctoral researchers from the course have gone on to very distinguished careers. Richard was very generous with his ideas and gave them out freely. His mathematical legacy is immense and can be felt all over the mathematical world. It is incumbent upon us at Columbia, who profited directly from his teaching and interactions with him, for his sake as well as our own, to contribute in the future to this legacy as much as we can.

Figure 6. Richard with Shing-Tung Yau.

A Poem in Memory of Richard Hamilton

Shing-Tung Yau

On hearing the news, I feel so grievous.

Dear Richard, the sudden grim news strikes like shattered jade, sunken pearls.

To nurture and then to destroy—what purpose has fate?

Through life and death, how deep my sorrow is!

A midsummer day in July, Richard, you came a long way to Tsinghua.

Rewarded and honored here, with further distinctions bestowed.

The theory of Ricci flow has grown into a vast subject.

Like gold sifted from the sands, your work has been recorded in mathematics history.

Wherever we were, the clouds and forest witnessed our good old days.

Looking back after these unforgettable years, white-haired gathered again.

We were happy to meet again and promised to each other that we should live up to one hundred years old.

Yet when I gaze at the heavens, who will share in my sorrow.

Young as we were, full of ambition when we first met in Cornell.

We discussed and brainstormed much by the lake.

The groundbreaking insights in differential geometry are exceedingly rare.

Big achievements will be reached if we drive the manifolds by the Ricci flow.

Chatting days and nights in nature when we met again in Princeton. A thousand-mile journey begins with a single step.

Focusing on sphere-like manifolds, you spurred forward with determination.

After a first success, your work was admired by the mathematics community.

By the shores of Southern California, we set high banners, guiding us toward greatness.

Distinguished scholars gather, and bright minds are drawn like arrows to the bow.

The conjectures and ideas of our predecessors, we seize the opportunity to tackle.

Singularities laid bare, their classification takes form.

Owls envy fledgling phoenixes, as winds sway the towering boughs.

The pleasant time during the banquet flew by so rapidly overnight.

You and I headed for New York and Cambridge. Oceans apart, we communicated less than before.

The topology of space was mysterious but attractive for us.

Loops deform into points, with no obstruction.

The Way is simple and pure, for the sphere is without peril.

Poincaré conjecture, suspended for a century, still looms.

Chinese legends of Jing Wei fill the sea with pebbles, and Yu Gong moves a mountain, stone by stone.

Intermediate success was achieved in three years, and you traveled north to visit me with joy and anticipation.

You presented to me your estimates, reflecting the spirit of the Li-Yau inequality. It was profound and unquestionably a masterpiece.

With better understanding of the singularities, the path revealed a subtle glow.

Hard as ever, we went through thick and thin wholeheartedly.

I invited you to visit Harvard. You gave a series of lectures.

With great admiration of how elegant the new theory is!

Spending a year at Harvard, we planned and deliberated together.

When curvature is uncontrollable, singularities must be classified.

However, if a singularity resembles a cigar, we can only lament: The Creator must have been drunk to make it so complicated.

From profound stillness to dynamic action, we crossed vast oceans time and again.

You surfed in daytime and attacked the problem at night. At your invitation, I visited your home for deep discussions.

Our estimates, though meticulous, still fell short of perfection!

At the new millennium, events began to stir. From afar came news—singularities could be controlled. With surgery, we handled each detail with care. After the operation, the flow resumed, growing easier, as if carried by the winds.

A great work from Russia, shrouded in secrecy, took years to complete.

The mathematical community was abuzz, celebrating two remarkable contributors.

The enigma of three-dimensional space was unraveled in a momentous breakthrough.

Honors followed, and achievements were inscribed in the annals of history.

The theory of flows flourished, profoundly expanding new domains of research.

Facing formidable challenges, no obstacle was insurmountable.

Ingenious ideas carved out unique paths, planting seeds for future marvels. Your extraordinary talent remains an eternal model for scholars.

We admired autumn leaves by the lake and strolled through the woods.

Battling ocean waves, we debated mathematics in quiet towns.

Our thoughts soared as friendships blossomed. But now, whoever could share those with me anymore?

Geometric analysis suddenly rose to prominence. You remained at the forefront through every twist and turn. Though the path was long and winding, we sailed together for four decades. Your joys became my happiness; your worries, my grief.

With support from afar, the great work reached completion.

We had hoped to build further upon this success, but illness struck without warning.

In an instant, you left, and all those concerns seemed inconsequential.

Who will now forge new paths? Who will take up these profound studies?

You cared little for fame or fortune, favoring a love for nature.

Now, in the heavenly realm, perhaps you converse with the sages of old.

Poincaré nods in approval, Ricci clasps your shoulder.

Your voice and appearance remain vivid, as if no time has passed; thoughts of you feel as fresh as ever.

The nature of things is to return to nothingness. Reflecting on our friendship fills me with sorrow.

I take up my pen, invoking your spirit.

Oh, soul, return!

Credits

Figure 1 is courtesy of Cynthia Hamilton.
Figure 2 is courtesy of Susan Harris.
Figure 3 is courtesy of International Congress of Basic Science.
Figure 4 is courtesy of Red Mahan.
Figure 5 is courtesy of Panagiota Daskalopoulos.
Figure 6 is courtesy of Shing-Tung Yau.